PAPER Special Section on Cryptography and Information Security

1-out-of-n Signatures from a Variety of Keys

Masayuki ABE†, Miyako OHKUBO†, *and* Koutarou SUZUKI†, *Members*

SUMMARY This paper addresses how to use public-keys of several different signature schemes to generate 1-out-of-n signatures. Previously known constructions are for either RSA-type keys only or DL-type keys only. We present a widely applicable method to construct a 1-out-of-n signature scheme that allows mixture use of different flavors of keys at the same time. The resulting scheme is more efficient than previous schemes even if it is used only with a single type of keys. With all DL-type keys, it yields shorter signatures than the ones of the previously known scheme based on the witness indistinguishable proofs by Cramer, et al. With all RSA-type keys, it reduces both computational and storage costs compared to that of the Ring signatures by Rivest, et al.

key words: signer ambiguity, ring signature, discrete-log

1. Introduction

A 1-out-of-n signature convinces a verifier that a document is signed by one of n possible independent signers without allowing the verifier to identify which signer it was. It can be seen as a simple group signature that has no group manager who can revoke the identity of the signer in case of emergency. Such a signature can also be seen as a kind of non-interactive proof that the signer owns a witness (secret-key) that corresponds to one of n commitments (public-keys) or theorems without leaking which one it really is. Such a primitive, as a signature scheme and/or a proof system, plays a central role in variety of applications such as group signatures [6], [9], designated verifier signatures [21], mix-nets [1], electronic voting [13], [14] and so on.

In [12], Cramer, Damgård and Shoenmakers presented a widely applicable, yet efficient construction of t-out-of-n witness indistinguishable proofs [17] based on secret sharing and public-coin honest verifier zero-knowledge proofs. It can be transformed into t-out-of-n signatures via the Fiat-Shamir technique [16]. It is especially suitable for converting Schnorr signatures [28] and Guillou-Quisquater signatures [20] into t-out-of-n signatures. It also allows to involve RSA signature scheme based on a zero-knowledge proof of knowledge about the factors of RSA modulus, e.g. [7], [8], but they are less efficient than the Schnorr or the GQ signatures both in computation and storage. [27] offers more intricate construction of t-out-of-n proofs for membership.

In [26], an efficient construction of 1-out-of-n signatures with RSA public-keys was introduced by Rivest, Shamir and Tauman. Called the Ring Signature Scheme, it is based on trapdoor-one-way permutations (TPs for short) and an ideal block cipher that is regarded as a perfectly random permutation. The name reflects its unique structure such that a signer who knows at least one witness (trapdoor information) can connect the head and tail of a series of n random permutations to shape the sequence into a ring. Since the trapdoor is essential in their construction, it is only for the keys like RSA's and the discrete-log keys are not supported.

There are other solutions that are more efficient but work only in non-separable models where all public-keys are related. For instance, when public-keys of the Schnorr signature scheme are chosen from a common group, one can construct an efficient 1-out-of-n signature scheme as shown in Appendix. Such non-separable but highly efficient schemes may be useful when used within a specific group. In general, however, public-keys are selected independently by each signer. Even key-length would differ from user to user. Constructions based on [12] and [26] suit a separable model where no underlying group are assumed. Hence, they are 'setup-free'; if one utilizes an existing public-key infrastructure, the key-setup phase only for this purpose is unnecessary. Furthermore, each key can be freely updated whenever each user wishes.

As introduced in [26], one application of 1-out-of-n signatures is to involve somebody else's public-keys into one's signature without their agreement. Although there are pros and cons for such usage, it is surely useful for protecting privacy. Unfortunately, all above mentioned known schemes have particular shortcomings for this purpose; What if one is using DSA while others are using RSA? Generating a new RSA key only for this purpose is not a great idea. It is important to have wide freedom for choosing various public-keys to involve.

Our contribution. We present a widely applicable method of constructing 1-out-of-n signature schemes that allows to use several flavors of public-keys such as these for integer factoring based schemes and discrete-

Manuscript received March 24, 2003.
Manuscript revised July 4, 2003.
Final manuscript received August 5, 2003.

†The authors are with NTT Information Sharing Platform Laboratories, NTT Corporation, Yokosuka-shi, 239-0847 Japan.

log based schemes at the same time. We describe two classes of signature schemes, which we call trapdoor-one-way type and three-move type, whose public-keys can be used for our construction.

Our approach also has several advantages even for the use with the same kind of keys like the former schemes:

1. When our scheme is used only with public-keys of three-move type signature schemes converted from zero-knowledge proof system, it results in a more efficient scheme than previously known three-move based construction [12] with regard to the size of signatures. For large n, it saves signature length about *by half.* Since this type of schemes includes the discrete-log based public-keys, this can be seen as the first construction of a ring signature scheme based on the discrete logarithm problem.
2. When our scheme is used only with the trapdoor-one-way based public-keys such as RSA, it results in a simplified ring signature scheme. By eliminating the use of block cipher and costly domain adjustment in the former scheme [26], our scheme offers shorter signature and less computation. In particular,

- The signature size of ours is about 13% less than that of the previous construction when RSA with modulus size 1024 bits are used.
- Both size of signature and computation in our signature generation is proportional to the *average* size of the modulus while that of former scheme it is proportional to the *maximum* size of the modulus. Accordingly, one long modulus does not impact efficiency in our scheme unlike the previous scheme.

We will show several concrete examples following our abstract construction. The security is proven in the random oracle model [2] as well as previously known schemes.

The rest of this paper is organized as follows. Section 2 defines security of 1-out-of-n signatures. We review two constructions that work in the separable model in Sect. 3. Section 4 describes our construction in an abstract way. Some concrete examples are given in Sect. 6. It includes a discrete-log version of the ring signature scheme, improved and simplified version of the RSA-based ring signatures, and small case of mixture use of RSA and DL type signatures. In Sect. 7 the efficiency of some concrete instantiations are analyzed in detail.

2. Definitions

Definition 2.1 (Signer Ambiguous Signature Schemes)

A 1-out-of-n signature scheme Sig is a triple of polynomial-time algorithms, $Sig = (\mathcal{G}, \mathcal{S}, \mathcal{V})$:
$(sk, pk) \leftarrow \mathcal{G}(1^\kappa, \mathsf{type})$, *which is a probabilistic algorithm that takes security parameter* κ *and a key-type specifier* type, *and outputs secret-key* sk *and public-key* pk.
$\sigma \leftarrow \mathcal{S}(L, sk, m)$, *which is a (probabilistic) algorithm that takes a set of public-keys* L, *a secret-key* sk *whose public-key is included in* L, *and a message* m, *and outputs a signature* σ.
$1/0 \leftarrow \mathcal{V}(L, m, \sigma)$, *which is an algorithm that takes* L, m *and signature* σ, *and outputs* 1 *or* 0 *meaning* accept *or* reject, *respectively. We require that* $\mathcal{V}(L, m, \mathcal{S}(L, sk, m)) = 1$ *for any message* m, *any* L *that consists of public-keys generated by* $\mathcal{G}$ *and any* sk *whose public-key is included in* L.

$\mathcal{G}$ can be seen as a collection of key generators for various signature schemes and type names one of them. If L includes public-keys based on different security parameters, the security of Sig is set to the smallest one among them.

The security of 1-out-of-n signature schemes has two aspects: *signer ambiguity* and *unforgeability.* Informally, the signer ambiguity is that it is infeasible to identify which secret-key is used to generate a signature.

Definition 2.2 (Signer Ambiguity) *Let* $L = \{pk_1, \ldots, pk_n\}$ *be the set of public-keys generated by* $(pk_i, sk_i) \leftarrow \mathcal{G}(1^{\kappa_i}, \mathsf{type}_i)$. *Sig is unconditionally signer ambiguous if, for any* L, m, sk, *and* $\sigma \leftarrow \mathcal{S}(L, sk, m)$, *given* (L, m, σ), *any unbound adversary identifies* $pk \in L$ *that corresponds to* sk *with probability exactly* $1/|L|$.

Unforgeability is defined similar to the notion of existential unforgeability against adaptive chosen message attacks [19]. To capture the situation that the signer involves public-keys generated by the adversary, we allow the adversary to add arbitrary public-keys to the list of public-keys and to ask the signing oracle to sign with an adaptively chosen subset of the keys. Accordingly, we consider *adaptive chosen message and chosen public-key attacks.* Formally, we define the security through the following game. Let (sk_i, pk_i) be a key of some type generated by $\mathcal{G}$, and let L_{init} be a collection of n such public-keys the adversary attacks. By κ we denote the smallest security parameter used to generate the keys in L_{init}. Let L_{add} be denote the list of public-keys added by the adversary, which is empty at the beginning. By L_{all}, we denote $L_{\mathsf{init}} \cup L_{\mathsf{add}}$.

Definition 2.3 (Adaptive Chosen Message and Chosen Public-key Attack)

1. *Adversary* $\mathcal{A}(L_{init})$ *makes the following queries arbitrary times in arbitrary order.*
   - $\mathcal{A}$ *sends* $Q_j = (L_j, m_j)$ *to the signing oracle* $\mathcal{SO}$. *If* $L_j \subseteq L_{all}$, *the oracle returns* σ_j *that*

satisfies $\mathcal{V}(L_j, m_j, \sigma_j) = 1$. *Otherwise,* $\perp$ *is returned.*

- $\mathcal{A}$ *issues* $\mathsf{add}(pk_j)$. *Public-key* pk_j *is then appended to* L_{add}.

2. $\mathcal{A}$ *outputs* $(\tilde{L}, \tilde{m}, \tilde{\sigma})$.

Let $\{(L_j, m_j, \sigma_j)\}$ *denote the history of conversation between* $\mathcal{SO}$ *and* $\mathcal{A}$. *We say that* $\mathcal{A}$ *wins the game if*

- $(\tilde{L}, \tilde{m}, \tilde{\sigma}) \notin \{(L_j, m_j, \sigma_j)\}$,
- $\tilde{L} \subseteq L_{init}$, *and*
- $\mathcal{V}(\tilde{L}, \tilde{m}, \tilde{\sigma}) = 1$.

The second condition is to prevent the trivial forgery that the adversary yields signatures with the self-generated public-keys.

Definition 2.4 (Unforgeability) *Sig is existentially unforgeable against adaptive chosen message and chosen public-key attacks if any probabilistic polynomial-time adversary* $\mathcal{A}$ *wins the above game with probability negligible in* κ.

It is important to remark that the above definition states that the list of public-keys must not be altered as well as the message. That is, one should not be able to add or remove public-keys associated to the given signatures.

3. Previous Schemes

3.1 Witness Indistinguishable Signatures [12]

Here we show a witness indistinguishable signature scheme in a concrete discrete logarithm setting. The scheme is based on [12] but slightly modified to work in separable setting, i.e. it accepts different groups. Let p_i, q_i be large primes. Let $\langle g_i \rangle$ denote a prime subgroup of $\mathbb{Z}_{p_i}^*$ generated by g_i whose order is q_i. Let x_i, y_i be $y_i = {g_i}^{x_i} \bmod p_i$. Here, x_i is the secret-key and (y_i, p_i, q_i, g_i) is the public-key. Let L be a set of (y_i, p_i, q_i, g_i) for $i = 1, \ldots, n$. We assume that all q_i are in the same size, say ℓ bits. Let $H : \{0,1\}^* \to \{0,1\}^{\ell-1}$ be a publicly available hash function.

A signer who owns secret-key x_k generates a signature for message m with public-key list L that includes his own public-key, in the following way.

W-1 (Simulation step): For every $i \in \{1, \ldots, n\} \setminus \{k\}$, compute $a_i = g_i^{s_i} y_i^{c_i} \bmod p_i$ for $s_i \leftarrow \mathbb{Z}_{q_i}$, $c_i \leftarrow \{0,1\}^{\ell-1}$.

W-2 (Real proof step): Compute

$$r_k \leftarrow \mathbb{Z}_{q_k}$$
$$a_k = {g_k}^{r_k} \bmod p_k$$
$$c = H(L, m, a_1, \cdots, a_n)$$
$$c_k = c \oplus c_1 \oplus \cdots \oplus c_{k-1} \oplus c_{k+1} \oplus \cdots \oplus c_n$$
($\oplus$: bitwise-XOR.)
$$s_k = r_k - c_k \cdot x_k \bmod q_k.$$

W-3 Output $\sigma = (c_1, s_1, \ldots, c_n, s_n)$.

A (L, m, σ) is valid if $s_i \in \mathbb{Z}_{q_i}$ and $c_i \in \{0,1\}^{\ell-1}$ for all $i = 1, \ldots, n$ and

$$\oplus_{i=1}^{n} c_i = H(L, m, g_1^{s_1} y_1^{c_1} \bmod p_1, \cdots, g_n^{s_n} y_n^{c_n} \bmod p_n).$$

The size of σ is at most $n(2\ell - 1)$ bits. The security can be proven in the random oracle model by using the rewinding simulation technique [18], [23], [25].

When $|q_i|$ differs a lot, a modification is needed to retain collision property. That is, ℓ is set to be larger than the largest $|q_i|$ and s_k is computed as $s_k = r_k - x_k \cdot CRH_k(c_k) \bmod q_k$ where $CRH_k : \{0,1\}^* \to \mathbb{Z}_{q_k}$ is a collision-resistant hash function.

3.2 Ring Signatures with Trapdoor-One-Way Permutations [26]

Let $f_i : \{0,1\}^\ell \to \{0,1\}^\ell$ be a trapdoor-one-way permutation where its inverse, f_i^{-1}, can be computed only if the trapdoor information is known. Let E, D be a symmetric-key encryption and decryption function whose message space is $\{0,1\}^\ell$. Let H be a hash function whose output domain matches to the key-space of E, D.

Given $f_1, \ldots, f_n$, the signer who can compute ${f_k}^{-1}$ generates a signature for message m as follows.

R-1 (Initialization): Compute $r_n = D_K(c_1)$ where $K = H(m)$ and $c_1 \leftarrow \{0,1\}^\ell$.

R-2 (Forward sequence): For $i = 1, \ldots, k-1$, compute $c_{i+1} = E_K(c_i \oplus f_i(s_i))$ for $s_i \leftarrow \{0,1\}^\ell$.

R-3 (Backward sequence): For $i = n, \ldots, k+1$, compute $r_{i-1} = D_K(r_i \oplus f_i(s_i))$ for $s_i \leftarrow \{0,1\}^\ell$.

R-4 (Shaping into a ring): Compute $s_k = f_k^{-1}(c_k \oplus r_k)$

The resulting signature is $(c_1, s_1, s_2, \ldots, s_n)$. A signature-message pair is verified by computing $K = H(m)$ and $c_{i+1} = E_K(c_i \oplus f_i(s_i))$ for $i = 1, \ldots, n$, and accept if $c_{n+1} = c_1$ holds.

In practice, each trapdoor permutations will be defined over individual domain such as $\mathbb{Z}_{N_i}$. In such a case, the above scheme need to transform such individual functions into common-domain trapdoor permutations. This transformation incurs some overhead. The following method is suggested in [26] to transform $\mathcal{E}_i$ into f_i defined over common-domain $\{0,1\}^\ell$ where $\ell = \max\{|N_i|\} + 160$. Let $\mathcal{E}_i$ be the RSA encryption function with modulus N_i. Let Q and S be positive integers such that $QN_i + S = s$ and $0 \le S < N_i$. Define

$$f_i(s) = \begin{cases} QN_i + \mathcal{E}_i(S) & \text{if } (Q+1)N_i \le 2^\ell \\ s & \text{otherwise.} \end{cases}$$

In order for the latter case to happen only with negligible probability, ℓ should be polynomially larger than the size of largest modulus. For instance, if the largest modulus is 2048 bits, ℓ will be 2048 + 160 bits. Accordingly, the resulting signature size is $2208(n+1)$ bits. This would be a large overhead when other moduli are all 1024 bits.

The above ring signature is existentially unforgeable against adaptive chosen message attacks in the ideal cipher model where E and D are modeled by truly random permutations.

3.3 Other Related Works

[5] extends the scheme of [26] to a threshold scheme with the cost of $O(2^t \log n)$ efficiency for threshold t. [22] considers deniable ring authentication that accepts variety of public-keys and a threshold of signers. It however, needs interaction between the signer and the verifier.

4. Our Scheme

4.1 Key Types

We consider two classes of signature schemes characterized by trapdoor-one-way permutations and three-move protocols, which we denote type-OW and type-3M, respectively.

Class type-OW includes schemes such as variants of RSA signatures [3], [10], Rabin signatures and Paillier signatures [24], which use trapdoor-one-way permutations. Let F be a one-way permutation and I be its inverse function defined over space C. Computing $F(pk, \cdot)$, $I(sk, \cdot)$ are easy but computing I without sk is infeasible as defined below.

Definition 4.1 (Trapdoor-one-way) *For any probabilistic polynomial-time algorithm $\mathcal{A}$, for $(pk, sk) \leftarrow \mathcal{G}(1^\kappa)$, and for $c \leftarrow \mathcal{C}$, $F(pk, \mathcal{A}(c, pk)) = c$ happens only with negligible probability in κ. Probability is taken over coin flips of $\mathcal{A}$, $\mathcal{G}$, and the choice of challenge c.*

Signature s is issued by inverting encoded message $e \in \mathcal{C}$ with secret-key sk; i.e., $s = I(sk, e)$, and verification is done by checking $e \stackrel{?}{=} F(pk, s)$, i.e., recovering the encoded message from signature s with public-key pk and comparing it to the given message.

Although trapdoor-one-way is sufficient to our construction, better security bound can be achieved if useful extra property is provided. Here, we assume that $F(pk, \cdot)$ is chosen from a family of claw-free permutations.

Definition 4.2 (Claw-free) *A family $\mathcal{F}_\kappa$ of pair of functions is claw-free family if, for any probabilistic polynomial-time algorithm $\mathcal{A}$, for $(f, f') \leftarrow \mathcal{F}_\kappa$, probability that, given (f, f'), $\mathcal{A}$ outputs (s, s') such that $f(s) = f'(s')$ only with negligible probability in κ. Probability is taken over coin flips of $\mathcal{A}$, and the choice of (f, f').*

A pair (s, s') that causes $f(s) = f'(s')$ is called a claw. For more formal treatment that states random sampling and other standard properties, please refer [19]. We consider that public-key pk generated by key-generator $\mathcal{G}(1^\kappa)$ uniformly selects a pair of claw-free function from $\mathcal{F}_\kappa$. Namely, we consider $f(\cdot) = F(pk, \cdot)$ and $f'(\cdot) = F'(pk, \cdot)$.

The RSA function is a concrete candidate of such f. For $f(s) = s^e \bmod n$, define $f'(s) = rs^e \bmod n$ where r is taken from $\mathbb{Z}_n^*$. Then, (f, f') is claw-free under the RSA assumption because a claw, say (s, s'), which satisfies $f(s) = s^e \bmod n = rs'^e \bmod n = f'(s')$, allows one to invert r as $r^{1/e} = s/s' \bmod n$. In general, claw-free permutations can be constructed from homomorphic or random-self-reducible trapdoor permutations [15].

Class type-3M, typified by the Schnorr signature scheme, includes the ones derived from three-move protocols, e.g., [11], which involve three polynomial-time algorithms, say A, Z and V performed by a prover and a verifier. The prover commits to $a \leftarrow A(sk; r)$ and answers to randomly chosen challenge c with $s = Z(sk, r, c)$. (By $A(sk; r)$, we denote that r is chosen uniformly from its proper domain and given to probabilistic algorithm A as a source of randomness. The domain may depend on sk as well as algorithm A. Similar notation will be used in the rest of paper.) The verifier accepts if $a = V(pk, s, c)$. The protocol must be honest verifier zero-knowledge. That is, for c and s randomly chosen from appropriate distribution, the distribution of $V(pk, s, c)$ is identical (or statistically close) to that of $A(sk; r)$ with uniformly chosen r. Due to technical reasons, we demand slightly more. That is, for every pk, sk and c, distribution of $V(pk, s, c)$ must be identical (or statistically close) to that of $A(sk; r)$ according to the uniform choice of s and r. Similarly, for every sk and c, $Z(sk, r, c)$ distributes uniformly (or statistically close to it) over its domain according to the uniform choice of r. Such property, which we call *fixed challenge uniformity*, is provided by many three-move protocols, e.g., $s = r + c \times sk$ distributes uniformly according to the uniform choice of r in Schnorr signature.

Additionally, the protocols must provide *collision property* defined as follows.

Definition 4.3 (Collision Property) *There exists a polynomial-time algorithm that computes sk from pk and two accepting conversations (a, c, s) and (a, c', s') where $(c, s) \neq (c', s')$.*

The Fiat-Shamir technique converts three-move protocols to secure digital signature schemes by generating

challenge c from a and message m via an ideal hash function. The resulting signature is (c, s) that can be verified by checking $c \stackrel{?}{=} hash(V(pk, c, s), m)$.

Some signature schemes are neither type-OW nor type-3M. For such schemes, we consider *compatibility* among signature schemes. Signature scheme A is compatible with scheme B if 1. A's secret-keys and public-keys can be used to issue and verify signatures of scheme B, and 2. breaking B (in the sense of existential unforgeable against adaptive chosen message attacks [19]) implies breaking A using the same key. For instance, DSS is not either type but it is compatible with the Schnorr signature scheme of type-3M. Since breaking the Schnorr signature scheme implies that the discrete-log is tractable with regard to the key, it implies DSS is broken, too. Thus, DSS keys can be involved in our scheme with type-3M.

With regard to type-OW schemes, however, special care may be needed. Remember that type-OW only shows the ability of computing $I(sk, \cdot)$ and does not necessarily imply possession of sk. Therefore, it is not sufficient that scheme A's keys can be used to scheme B, but it has to be true that ability of computing $I(sk, \cdot)$ of scheme B is sufficient to generate signatures of A.

The signature scheme in [4] is an interesting scheme that belongs to type-OW but its keys are also compatible with type-3M ones. In such a case, one can involve the keys as either type-OW or type-3M keys according to ones convenience (perhaps by comparing their computation and communication costs which depend on particular implementation).

4.2 Description

[Key Generation]

A signer generates his own key pair by using the signature generation function of a signature scheme of his choice: $(sk, pk) \leftarrow \mathcal{G}(1^\kappa, \text{type})$

[Public-key Listing]

Compose $L = \{pk_1, \ldots, pk_n\}$, which is a list of public-keys where the signer's public-key is located as pk_k. (Corresponding secret-key is referred as sk_k hereafter.)

We assume that L is sorted so that the first ℓ keys are of type-OW and the others are of type-3M. This sorting is only for exposition and irrelevant to user ambiguity.

For $a, b \in C_i$, let $a+b$ denote the group operation of Abelian group C_i and $a-b$ be the group operation with inverse of b. These binary operators are used without subscripts that denotes each group. Let $H_i : \{0,1\}^* \rightarrow C_i$ be a hash function. Domain C_i depends on pk_i. For (pk_i, sk_i) of type-OW, C_i corresponds to the domain where $F(pk_i, \cdot)$ and $I(sk_i, \cdot)$ are defined. For (pk_i, sk_i) of type-3M, it corresponds to the randomness space of $A_i(sk_i; r)$.

[Signature Generation]

To simplify the description, we define algorithms A, V, Z for type-OW keys as follows.

$$A(sk; r) \stackrel{\text{def}}{=} r$$
$$V(pk, s, c) \stackrel{\text{def}}{=} c + F(pk, s)$$
$$Z(sk, r, c) \stackrel{\text{def}}{=} I(sk, r - c)$$

In this way, A, V, Z can be treated just like those of type-3M. It is easily verified that they provide fixed challenge uniformity, which is necessary to prove the signer ambiguity. In the following, the indices written as subscripts are considered within $\{1, \cdots, n\}$. Especially, $n+1$ is equivalent to 1.

G-1 (Initialization): Compute $a_k = A_k(sk_k; r)$ and $c_{k+1} = H_{k+1}(L, m, a_k)$.

G-2 (Forward sequence): For $i = k+1, .., n, 1, ..., k-1$, select s_i randomly and compute

$$a_i = V_i(pk_i, s_i, c_i), \text{ and}$$
$$c_{i+1} = H_{i+1}(L, m, a_i).$$

G-3 (Forming the ring): Compute $s_k = Z_k(sk_k, r, c_k)$.

G-4 Output $\sigma = (c_1, s_1, s_2, \cdots, s_n)$.

[Signature Verification]

Given (L, m, σ), purse σ as above and compute the following. For $i = 1, \cdots, n$, compute

$$a_i = V_i(pk_i, s_i, c_i),$$
$$c_{i+1} = H_{i+1}(L, m, a_i).$$

Accept if c_{n+1} computed at the last moment of the above loop is identical to the input c_1. Reject otherwise.

5. Security

Theorem 5.1 *The proposed scheme is unconditionally signer-ambiguous.*

Proof. We show that distribution of signature σ is independent of k. Observe that for every $i \in \{1, \ldots, n\}$ except for $i = k$, s_i is uniformly chosen from appropriate domain. For $i = k$, s_k also distributes uniformly due to the uniform choice of r and the fixed challenge uniformity of Z. Next, we consider the distribution of c_1. Observe that c_1 distributes according to a_n. If $k \neq n$, a_n distributes as if it were generated by $A_n(sk_n; r_n)$ with uniformly chosen r_n due to the uniform choice of s_n and the fixed challenge uniformity of V. Otherwise,

if $k = n$, a_n is indeed generated by $A_n(sk_n; r)$. Accordingly, distribution of c_1 is independent of k. □

Note that the signer ambiguity does not rely on ideal randomness assumption on the hash function. Next, we claim unforgeability.

Theorem 5.2 (Unforgeability) *The proposed scheme is existentially unforgeable against adaptive chosen message and chosen public-key attacks.*

Proof. Let $\mathcal{A}$ be $(\epsilon, \tau, q_s, q_h, q_a)$-adversary that wins the game defined in Definition 2.3, with a forged signature $(\tilde{L}, \tilde{m}, \tilde{c}_1, \tilde{s}_1, \ldots, \tilde{s}_{|\tilde{L}|})$ with probability at least ϵ, running time at most τ, at most q_s signature requests, q_a add requests. Here, q_h denotes the upper bound of random oracle calls made during the game, i.e., calls from the adversary, from the signing oracle, and for verification of the forged signature. For each $F_i(pk_i, \cdot)$, let $F_i'(pk_i, \cdot)$ be a partner of claw-free permutations as defined in Definition 4.2. We prove the theorem by proving the following lemma, which is the quantitive version of the theorem.

Lemma 5.3 *If there exists $(\epsilon, \tau, q_s, q_h, q_a)$-adversary $\mathcal{A}$, then, there exists (ϵ', τ')-simulator $\mathcal{SIM}$ that takes $(L_{\mathsf{init}}, F_1, F_1', \ldots, F_\ell, F_\ell')$ as an input and finds a secret-key of type-3M in L_{init} or a claw for one of the claw-free permutations, with probability ϵ' and running time τ' where*

$$\epsilon' > \frac{9}{50}, \quad \tau' < \frac{q_h}{\epsilon}\left(\frac{1}{(1-\frac{q_h}{q})^{q_s}} + \frac{2}{(1-\frac{q_h}{q})^{q_s+1}}\right)\tau$$

Proof. Let τ and ϵ be the running time and success probability of $\mathcal{A}$. Let q_s be the maximum number of signing oracle queries made by $\mathcal{A}$. Let q_h be the maximum number of hash queries made during the attack. Note that q_h includes the ones from $\mathcal{SIM}$ for signing oracle simulation, verification, and etc. Namely, q_h denotes the total length of the tables that defines input/output correspondence of the random oracles. By q, we denote the size of smallest C_i (regardless of the types of keys) defined by L_{init}.

We first show that, by treating all hash functions as random oracles, signatures can be generated without knowing any secret-key at all. Let (L_j, m_j) be a query to the signing oracle and let n_j denote $|L_j|$. In the following, pk_i means the i-th public-key stored in L_j and all functions and variables accompanied by suffix i are defined relative to pk_i.

[Signing Oracle Simulation]

S-1. Choose c_1, s_1 randomly and compute $a_1 = V_1(pk_1, s_1, c_1)$.

S-2. For $i = 2, \cdots, n_j$, choose s_i randomly and compute

$$c_i = H_i(L_j, m_j, a_{i-1}),$$
$$a_i = V_i(pk_i, s_i, c_i).$$

S-3. Assign c_1 to the value of $H_1(L_j, m_j, a_{n_j})$.

S-4. Output $\sigma_j = (c_1, s_1, \ldots, s_{n_j})$.

Observe that back-patching of hash value is done only once for H_1 at step 3 to simulate one signature. It fails only if exactly the same query (L_j, m_j, a_{n_j}) has been already made to H_1. Since a_{n_j} distributes uniformly due to the fixed-challenge uniformity of V_i and uniform choice of s_{n_j} in step 2, such a_{n_j} is included in the past $< q_h$ queries to H_1 with probability at most $\frac{q_h}{q}$. Thus, step 3 is successful with probability at least $1 - \frac{q_h}{q}$ and q_s signatures are successfully simulated with probability at least $(1 - \frac{q_h}{q})^{q_s}$. Except for such negligible failure cases, the simulated signatures distribute as well as the real ones. Since q is supposed to be exponentially large while other parameters in the above formula are polynomially bound, the simulated signature are statistically close to those in the real run.

We proceed to construction of $\mathcal{SIM}$. For simplicity, we wrap the random oracles $H_1, H_2, \ldots$ to a single random oracle whose j-th query is formatted as $Q_j = (pk, L_j, m_j, a_j)$. If $pk = pk_i \in L_{\mathsf{all}}$, the unified random oracle returns a random value that defines $H_i(L_j, m_j, a_j)$. (Note that, since pk is used only as a label to identify individual hash function H_i, it does not matter even if $pk \notin L_j$ and/or $L_j \not\subseteq L_{\mathsf{all}}$.) If $pk \notin L_{\mathsf{all}}$, it returns $\perp$ (since no hash function is defined for such public-key). It returns the same value for duplicated queries. (We do not explicitly describe such ineffective interactions in the following for conciseness.)

A key observation is the following: Suppose that $\mathcal{A}$ outputs forged signature $(\tilde{L}, \tilde{m}, \tilde{c}_1, \tilde{s}_1, \ldots, \tilde{s}_{|\tilde{L}|})$. Then, for each $pk_i \in \tilde{L}$ (from now, pk_i denotes i-th public-key in $\tilde{L}$ and all functions and variables with suffix i are defined relative to pk_i), random oracle query $(pk_i, \tilde{L}, \tilde{m}, V_i(pk_i, \tilde{s}_i, \tilde{c}_i))$ must be made. Since these hash values form a ring, there exist $i' \in \{1, \ldots, |\tilde{L}|\}$ and a pair of indices (u, v) such that $Q_u = (pk_{i'+1}, \tilde{L}, \tilde{m}, a_u)$, $Q_v = (pk_{i'}, \tilde{L}, \tilde{m}, a_v)$ and $u \le v$. Namely, $pk_{i'}$ lies in between the gap of hash query order. We call such (u, v) a gap index pair. Note that $u = v$ happens only if the resulting $\tilde{L}$ contains only one public-key. If there are two or more gap indices with regard to the signature, only the smallest one is considered.

Given L_{init} and pairs of claw-free permutations as described in Lemma 5.3, $\mathcal{SIM}$ works as follows.

A-1. Choose an index $u \in \{1, \ldots, q_h\}$ (expecting that u is to be the smaller one of the gap index pair (u, v)).

A-2. Run $\mathcal{A}$ and simulates the environment of the

game defined in Definition 2.3 as follows. For every signing oracle query, return a simulated signature as mentioned above. For every random oracle query, return a random value *except for the following case;* Let $pk_i^\star$ be the public-key specified by the u-th query Q_u. If $pk_i^\star$ is of type-OW, for every j-th query to H_i made after Q_u, choose $s_{ij}^\star$ randomly and return $a_u - F'(pk_i^\star, s_{ij}^\star)$. For each add request, simply add the public-key to the list.

A-3. $\mathcal{A}$ outputs $(\tilde{L}, \tilde{m}, \tilde{c}_1, \tilde{s}_1, \ldots, \tilde{s}_{|\tilde{L}|})$ that satisfies all winning conditions of the forgery game and corresponds to gap index (u, v) for some $v \geq u$. Otherwise, restart from A-1.

B-1. If i'-th public-key in $\tilde{L}$ is $pk_i^\star$ and it is of type-OW, it holds that $a_u = H_i(\tilde{L}, \tilde{m}, a_v) + F(pk_i^\star, \tilde{s}_{i'})$ as defined by verification procedure. Since $H_i(\tilde{L}, \tilde{m}, a_v) = a_u - F'(pk_i^\star, s_{ij}^\star)$ is set in step A-2, we have $F(pk_i^\star, \tilde{s}_{i'}) = F'(pk_i^\star, s_{ij}^\star)$, which is a claw. $\mathcal{SIM}$ thus completes the simulation by outputting claw $(\tilde{s}_{i'}, s_{ij}^\star)$. If $pk_i^\star$ is type-3M, otherwise, $\mathcal{SIM}$ proceeds to phase C.

C-1. Restart $\mathcal{A}$ handling its queries exactly in the same way as done in phase A making exactly the same random choices until Q_u is made. For Q_u and all subsequent queries, simulation proceeds in the same way as phase A but using a new independent random choices.

C-2. $\mathcal{A}$ outputs $(\tilde{L}, \tilde{m}, \tilde{c}_1', \tilde{s}_1', \ldots, \tilde{s}_{|\tilde{L}|}')$ as well as step A-3 that corresponds to gap index (u, v') for some $v' \geq u$. Let c and c' be replies to Q_v in phase A and $Q_{v'}$ in phase C respectively. If $c \neq c'$, proceed to phase D. Otherwise, restart from C-1.

D-1. $\mathcal{SIM}$ now extracts $sk_i^\star$ using $(a_u, c, \tilde{s}_{i'})$ and $(a_u, c', \tilde{s}_{i'}')$, that forms a collision with regard to $pk_i^\star$. $\mathcal{SIM}$ outputs $(sk_i^\star, pk_i^\star)$ and stops.

Now we evaluate the above reduction. Let ϵ_1 denote the probability that the expected event in A-3 happens. We then have

$$\epsilon_1 > \frac{1}{q_h}\left(1 - \frac{q_h}{q}\right)^{q_s} \epsilon, \tag{1}$$

where factor $\frac{1}{q_h}$ is for the cost of guessing correct index u, and factor $(1 - \frac{q_h}{q})^{q_s}$ is for the signing oracle simulation. By repeating phase A at most $\tau_1 = 1/\epsilon_1$ times, the event at A-3 happens with probability $1 - (1 - \epsilon_1)^{\frac{1}{\epsilon_1}} > 1 - \exp(-1) > 3/5$.

If L_{init} contains only type-OW keys, $\mathcal{SIM}$ does find a claw in B-1. In such a case, $\mathcal{SIM}$ runs in time τ_1 and success probability at least 3/5. Except for such a special case, we proceed evaluation assuming that the favorable result in B-1 does not come out.

To evaluate phase C, we apply classical heavy-row argument [18]. Consider a binary matrix whose row corresponds to the random choices (including the entire random tape of $\mathcal{A}$) made before query Q_u is answered, and whose column corresponds to the random choices after that. Each cell contains 1 if $\mathcal{A}$ causes the expected event of A-3. Otherwise it contains 0. The probability that a randomly chosen cell contains 1 is ϵ_1. A heavy row is a row that contains 1 more than $\epsilon_1/2$ of its cells. Now, due to the heavy-row lemma, the random choices made during the successful run in phase A is in a heavy row with probability at least 1/2. Therefore, the successful case in C-2 happens with probability at least $\epsilon_1/2$. Besides the probability that random oracle query Q_v in phase A and $Q_{v'}$ in phase C results in an identical hash value is bound by q_h/q. (Note that this is not $1/q$ because there are at most $q_h - u + 1$ candidates of queries that forms a gap index in combination with u and it is up to the adversary which one to take. Accordingly, we need to bound this probability by the probability that the value replied to Q_v in phase A also appears in q_h answers of phase C.) Therefore, the total successful case in C is $\frac{\epsilon_1}{2}(1 - \frac{q_h}{q})$.

By repeating phase C at most $\tau_2 = \frac{2}{(1-\frac{q_h}{q})\epsilon_1}$ times, the event at C happens with probability greater than 3/5 as well as phase A.

Finally, we analyse phase D. If $c \neq c'$, $(a_u, c, \tilde{s}_{i'})$ and $(a_u, c', \tilde{s}_{i'}')$ forms a collision with regard to $pk_i^\star (= pk_{i'})$, and the simulator can compute secret-key $sk_i^\star$ due to the collision property. Since $\tilde{L} \subseteq L_{\mathsf{init}}$ should hold by the wining condition of the game, $pk_i^\star$ is included in L_{init}. Thus the simulator is successful in computing a secret-key that corresponds to one of the public-keys in L_{init}.

The total running time is upper bound by

$$\begin{aligned}\tau_1 + \tau_2 &= \left(\frac{1}{\epsilon_1} + \frac{2}{(1 - \frac{q_h}{q})\epsilon_1}\right)\tau \\ &< \frac{q_h}{\epsilon}\left(\frac{1}{(1 - \frac{q_h}{q})q_s} + \frac{2}{(1 - \frac{q_h}{q})q_s + 1}\right)\tau.\end{aligned}$$

When $q_s q_h \ll q$, the central term of the above formula is very close to 3, and the running time $\tau_1 + \tau_2$ is approximately $\frac{3q_h}{\epsilon}$, assuming that the running time of $\mathcal{A}$ is the dominant factor compared to other operations such as A, V, F and hashing. The success probability is at least

$$\frac{3}{5} \cdot \frac{1}{2} \cdot \frac{3}{5} = \frac{9}{50}$$

Therefore, if q is exponential, ϵ is not negligible, and q_h and q_s are bound by polynomial in the security parameter, $\mathcal{SIM}$ runs in polynomial-time with sufficiently large success probability. □

Due to the lemma 5.3, Theorem 5.2 can be proven.

Because if there exist $\mathcal{SIM}$ that runs in polynomial-time, it allows one to compute a claw for at least one claw-free family or extracts at least one secret-key that corresponds to type-3M public-key in L_{init}. It contradicts the assumptions. □

6. Concrete Examples

6.1 All Discrete-Log Case

For $i = 1, \ldots, n$, let (y_i, p_i, q_i, g_i) be DL public-keys as described in Sect. 3.1 and $H_i : \{0,1\}^* \to \mathbb{Z}_{q_i}$ be hash functions. Let L be a list of these public-keys. A signer who has secret-key x_k generates a signature for message m as follows.

[Signature Generation]

D-1 (Initialization): Select $r \leftarrow \mathbb{Z}_{q_k}$ and compute $c_{k+1} = H_{k+1}(L, m, g_k{}^r \bmod p_k)$.

D-2 (Forward sequence): For $i = k+1, .., n, 1, ..., k-1$, select $s_i \leftarrow \mathbb{Z}_{q_i}$ and compute $c_{i+1} = H_{i+1}(L, m, g_i^{s_i} y_i^{c_i} \bmod p_i)$.

D-3 (Forming the ring): Compute $s_k = r - x_k c_k \bmod q_k$.

The resulting signature for m and L is $(c_1, s_1, s_2, \cdots, s_n)$.

[Signature Verification]

For $i = 1, \ldots, n$, compute $a_i = g_i^{s_i} y_i^{c_i} \bmod p_i$ and $c_{i+1} = H_{i+1}(L, m, a_i)$ where subscript $n+1$ is translated to 1. Accept if the c_1 computed at the last step equals the input c_1. Reject otherwise.

Intuitively, this scheme is a ring of the Schnorr signatures where each challenge is taken from the previous step. Indeed, it is the Schnorr signature scheme when $n = 1$.

6.2 All RSA Case

For $i = 1, \ldots, n$, let (e_i, N_i) be RSA public-keys and $H_i : \{0,1\}^* \to \mathbb{Z}_{N_i}$ be hash functions. Let L be a list of these public-keys. A signer who has secret-key d_k generates a signature for message m as follows.

[Signature generation]

T-1 (Initialization): Select $r_k \leftarrow \mathbb{Z}_{N_k}$ and compute $c_{k+1} = H_{k+1}(L, m, r_k)$.

T-2 (Forward sequence): For $i = k+1, .., n, 1, ..., k-1$, select $s_i \leftarrow \mathbb{Z}_{N_i}$, and compute $c_{i+1} = H_{i+1}(L, m, c_i + s_i^{e_i} \bmod N_i)$.

T-3 (Shaping into a ring): Compute $s_k = (r_k - c_k)^{d_k} \bmod N_k$

The resulting signature for m and L is $(c_1, s_1, s_2, \ldots, s_n)$.

[Signature Verification]

For $i = 1, \ldots, n$, compute $r_i = c_i + s_i^{e_i} \bmod N_i$ and then $c_{i+1} = H_{i+1}(L, m, r_i)$. Accept if c_{n+1} computed at the last moment of the above loop is identical to the input c_1. Reject otherwise.

6.3 Mixture Case: RSA and Schnorr

We finally show a small example for involving both RSA-type and DL-type keys. For simplicity, we consider the case $n = 2$, i.e., only two public-keys are involved. Let L consist of RSA public-key (e, N) and one Schnorr public-key (y, g, p, q). Let $H_1 : \{0,1\}^* \to \mathbb{Z}_N$ and $H_2 : \{0,1\}^* \to \mathbb{Z}_q$ be hash functions. A signer who has the RSA-type secret-key, d, generates a signature for message m as follows.

[Signature generation]

M-1 (Initialization): Select $r \leftarrow \mathbb{Z}_N$ and compute $c_2 = H_2(L, m, r)$.

M-2 (Forward sequence): Select $s_2 \leftarrow \mathbb{Z}_q$ and compute $c_1 = H_1(L, m, g^{s_2} y^{c_2} \bmod p)$.

M-3 (Shaping into a ring): Compute $s_1 = (r - c_1)^d \bmod N$

The resulting signature is (c_1, s_1, s_2).

[Signature Verification]

Given (L, m, c_1, s_1, s_2), compute $c_2 = H_2(L, m, c_1 + s_1^e \bmod N)$. Accept if $c_1 = H_1(L, m, g^{s_2} y^{c_2} \bmod p)$. Reject, otherwise.

The signature can be shorten by selecting (c_2, s_1, s_2) as a signature because $|c_1|$ is the size of RSA modulus typically > 1024 bits while $|c_2|$ is the size of q typically > 160 bits.

7. Efficiency

We compare our ring signature scheme with the existing schemes using DL, ECDL (elliptic curve DL) and RSA trapdoor functions, in terms of the length of a signature and the computational cost of signature generation and verification. We refer the scheme in Sect. 3.1 by "WI signatures" and the scheme in Sect. 3.2 with RSA trapdoor function by "RSA ring signatures," hereafter. Let n be the number of signers of ring signature.

Table 1 shows the comparison in terms of the length of the signature. Here, L(DL) is the length of exponent of DL signature, and is typically 160-bit. L(RSA) is the length of modulus of RSA signature, and is typically 1024-bit. L(EC) is the length of the size of cyclic subgroup in elliptic curve, and is typically 160-bit. From the table, we can see that the length of our

Table 1 Comparison of the length of signatrues (bits).

	Length of Signature	Typical value
WI signature	$(L(\mathrm{DL})+L(\mathrm{DL}))\times n$	$320\times n$
Ours (DL)	$L(\mathrm{DL})+L(\mathrm{DL})\times n$	$160+160\times n$
Ours (ECDL)	$L(\mathrm{EC})+L(\mathrm{EC})\times n$	$160+160\times n$
RSA ring signature	$(L(\mathrm{RSA})+160)$ $+(L(\mathrm{RSA})+160)\times n$	$1184+1184\times n$
Ours (RSA)	$L(\mathrm{RSA})+L(\mathrm{RSA})\times n$	$1024+1024\times n$

Table 2 Computational costs for signature generation and verification, and their typical value.

	Signing Cost	Typical value
WI signature	$T(\mathrm{DL})\times 5/4\times n$	$2.0\times 10^8\times n$
Ours (DL)	$T(\mathrm{DL})\times 5/4\times n$	$2.0\times 10^8\times n$
Ours (ECDL)	$T(\mathrm{EC})\times 5/4\times n$	$7.1\times 10^7\times n$
RSA ring signature	$T(\mathrm{RSA}^{-1})$ $+T(\mathrm{RSA})\times n$	1.0×10^9 $+1.6\times 10^7\times n$
Ours (RSA)	$T(\mathrm{RSA}^{-1})$ $+T(\mathrm{RSA})\times n$	1.0×10^9 $+1.6\times 10^7\times n$
	Verification Cost	**Typical value**
WI signature	$T(\mathrm{DL})\times 5/4\times n$	$2.0\times 10^8\times n$
Ours (DL)	$T(\mathrm{DL})\times 5/4\times n$	$2.0\times 10^8\times n$
Ours (ECDL)	$T(\mathrm{EC})\times 5/4\times n$	$7.1\times 10^7\times n$
RSA ring sig.	$T(\mathrm{RSA})\times n$	$1.6\times 10^7\times n$
Ours (RSA)	$T(\mathrm{RSA})\times n$	$1.6\times 10^7\times n$

signature with DL is *one half* of WI signature for large n, and that the length of our signature with RSA is 13% less than the length of RSA ring signature.

Table 2 shows the comparison in terms of the computational costs of signature generation and verification. Here, $T(DL)$, $T(EC)$, $T(RSA^{-1})$ and $T(RSA)$ are the computational costs of modular exponentiation, scalar multiplication on elliptic curve, inverse RSA function and RSA function, respectively. Typically, $T(\mathrm{DL})=T((1024)^{(160)})$, $T(\mathrm{EC})=T((160)\cdot(EC160))$, $T(\mathrm{RSA}^{-1})=T((1024)^{(1024)})$ and $T(\mathrm{RSA})=T((1024)^{(16)})$. Here, $T((x)^{(y)})$ is the number of (single precision) arithmetic operation of exponentiation with x-bit modulus and y-bit exponent, and is estimated $x^2\times y$. Exponentiation with y-bit exponent needs y x-bit multiplications, using binary method and the fact costs of square is half of multiplication. x-bit multiplication needs x^2 (single precision) arithmetic operations. $T((y)\cdot(ECx))$ is the number of (single precision) arithmetic operation of scalar multiplication on elliptic curve with x-bit base field and y-bit scalar, and is estimated $x^2\times 14\times y$. Scalar multiplication on elliptic curve with y-bit scalar needs y additions of points, using binary method and the fact costs of doubling is half of costs of addition. Addition of points with x-bit base field needs 14 x-bit multiplications, using Jacobian coordinate. x-bit multiplication needs x^2 (single precision) arithmetic operations. Hence, we have

$$T((1024)^{(1024)})=1024^2\times 1024\approx 1.07\times 10^9,$$
$$T((1024)^{(160)})=1024^2\times 160\approx 1.67\times 10^8,$$
$$T((1024)^{(16)})=1024^2\times 16\approx 1.67\times 10^7,$$
$$T((160)\cdot(EC160))=160^2\times 14\times 160\approx 5.73\times 10^7.$$

The computational costs of exponentiation with two basis is 5/4 of exponentiation with single basis, using two basis binary method. From these tables, we can see that the computational costs of our signature with DL is as same as WI signature, and that the computational costs of our signature with RSA is as same as RSA ring signature.

Notice that in known schemes the length and the computational costs of signature is proportional to the *maximum* of the length of DL exponent / RSA modulus. In our scheme, the length and the computational costs of signature is proportional to the *average* of the length of DL exponent / RSA modulus, since our scheme need not to round up the length to the maximum length.

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Appendix: An Efficient Scheme in Non-separable Model

Let p, q be large primes. Let $\langle g \rangle$ denote a prime subgroup in $\mathbb{Z}_p^*$ generated by g whose order is q. Let x_i, y_i be $y_i = g^{x_i} \bmod p$. Here x_i is the secret-key and (y_i, p, q, g) is the public-key. All member use common p, q, g in the non-separable model. So only y_i is different in public-keys for each member. Let L be a set of (y_i, p, q, g) for $i = 1, \ldots, n$. Let $H : \{0,1\}^* \to \mathbb{Z}_q$ be a hash function.

A signer who owns secret-key x_k generates a signature for message m with public-key list L that includes y_k, in the following way.

S-1 Select $\alpha, c_i \leftarrow \mathbb{Z}_q$ for $i = 1, \ldots, n$, $i \neq k$, and compute $a = g^{\alpha} \prod_{i=1, i \neq k}^{n} y_i^{c_i} \bmod p$.

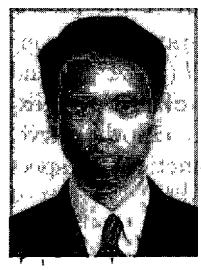
S-2 Compute

$$c = H(L, m, a)$$
$$c_k = c - (c_1 + \cdots + c_{k-1} + c_{k+1} + \cdots + c_n) \bmod q$$
$$s = \alpha - c_k \cdot x_k \bmod q.$$

The resulting signature is $\sigma = (s, c_1, \ldots, c_n)$. (L, m, σ) is valid if

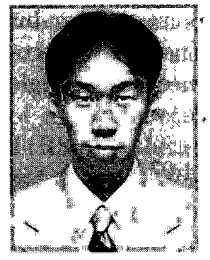
$$\sum_{i=1}^{n} c_i \equiv H(L, m, g^s y_1^{c_1} \cdots y_n^{c_n}) \bmod q.$$

The security of this scheme can be reduced to the discrete-log problem (representation problem) by standard rewinding simulation.



Masayuki Abe received the B.S., M.S. degrees from Science University of Tokyo in 1990 and 1992, respectively, and Ph.D. degree from the University of Tokyo in 2002. He is a senior research scientist of NTT Information Sharing Platform Laboratories. Member of IACR.

Miyako Ohkubo received the M.E. degree from Shinshu University in 1997. She is a researcher of NTT Information Sharing Platform Laboratories and a Ph.D. student of Chuo University. She received the SCIS Paper Award in 2000. Member of IACR.



Koutarou Suzuki received the B.S., M.S., and Ph.D. degrees from the University of Tokyo in 1994, 1996, and 1999, respectively. He is a researcher of NTT Information Sharing Platform Laboratories. He received the SCIS Paper Award in 2002. Member of IACR.